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Schwinger-Dyson and Large N_c Loop Equation for Supersymmetric Yang-Mills Theory [1]

H. Itoyama
and
H. Takashino

Department of Physics,
Faculty of Science, Osaka University,
Toyonaka, Osaka, 560 Japan

Abstract

We derive an infinite sequence of Schwinger-Dyson equations for $N = 1$ supersymmetric Yang-Mills theory. The fundamental and the only variable employed is the Wilson-loop geometrically represented in $N = 1$ superspace: it organizes an infinite number of supersymmetrizing insertions into the ordinary Wilson-loop as a single entity. In the large N_c limit, our equation becomes a closed loop equation for the one-point function of the Wilson-loop average.

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A number of recent developments on nonperturbative phenomena associated with supersymmetric gauge theories suggest that old questions such as quark confinement, monopole condensation and dynamical symmetry breaking can be understood better when supersymmetry is operating. In years to come, these developments may even be extended to physics of superstrings to which nonperturbative formulation is still largely lacking.

Schwinger-Dyson approach is a formulation which has been successful in deriving exact results in matrix models describing toy models of noncritical strings. It has supplied some valuable results in more realistic theories with manageable approximations. Clearly the most transparent formulation of the Schwinger-Dyson equations in gauge theories is given in terms of the gauge invariant Wilson-loop variable. It is well-known that one can write down the infinite sequence of equations using the geometric operations alone acting on the loop in pure Yang-Mills theory. In the large N_c limit, it reduces to a closed loop equation for the one-point function of the Wilson-loop average [1]. In this letter, we will provide its $N=1$ supersymmetric counterpart.[2]

Despite its obvious interest and relevance to the motivations decades ago and some current issues, this problem we will address ourselves below has neither been thoroughly investigated nor completed in the literature. See [2], [3] for earlier references. The fundamental variable we will employ is the gauge invariant Wilson-loop which is defined in terms of the path-ordered exponential in $N=1$ superspace [4],[5],[6].

Let us briefly recall the case of ordinary Yang-Mills theory: $\mathcal{L}_{YM} = \frac{(ig)^2}{4kg^2} tr v_{mn} v^{mn}$, $igv_{mn} \equiv [D_m, D_n]$, $D_m \equiv \partial_m + igv_m$, $v_m \equiv v_m^{(r)} T^r$, $trT^rT^s = k\delta^{rs}$. We take the gauge group to be $U(N_c)$ for simplicity. We denote by $\mathcal{C}_{xy}$ a path which begins with x and ends at y. Let $\mathbf{W}[\mathcal{C}] \equiv << \frac{k}{N_c} trU_{\mathcal{C}} >>$ with $U_{\mathcal{C}} \equiv P\exp(-ig\oint_{\mathcal{C}} dz^m v_m(z))$ be the one-point function of the Wilson-loop average associated with any closed loop $\mathcal{C}$ in Minkowski space. A fundamental role is played by the area derivative acting on the Wilson-loop, which produces the field strength. The subsequent action of the ordinary derivative supplies the equation of motion of Yang-Mills theory:

$$\frac{\partial}{\partial x_\ell} \frac{\partial}{\partial \Sigma^{m\ell}(x)} \mathbf{W}[\mathcal{C}] = -ig << \frac{k}{N_c} trD^\ell v_{m\ell}(x) U_{\mathcal{C}_{xx}} >> \quad . \tag{1}$$

[2]Matter supermultiplets can be added to produce another term in the Schwinger-Dyson equations.

We remind the readers that the area derivative acting on a functional $f[\mathcal{C}_{yz}]$ can be defined through an increment associated with an addition of an infinitesimal area element at the point $x^m = x^m(s,t)$ on the path $\mathcal{C}_{yz}$:

$$\begin{aligned} \Delta f[\mathcal{C}_{yz}] &\equiv f[\mathcal{C}_{yz} + \delta\mathcal{C}_x^{(1)}] - f[\mathcal{C}_{yz} + \delta\mathcal{C}_x^{(2)}] \\ &\equiv \frac{\partial f[\mathcal{C}_{yz}]}{\partial \Sigma^{m\ell}(x)} \frac{1}{2} \left(\frac{\partial x^m}{\partial s}\frac{\partial x^\ell}{\partial t} - \frac{\partial x^m}{\partial t}\frac{\partial x^\ell}{\partial s} \right) \Delta s \Delta t \\ &\equiv \frac{\partial f[\mathcal{C}_{yz}]}{\partial \Sigma^{m\ell}(x)} \left(\Delta\Sigma\right)^{m\ell}(x; \Delta s, \Delta t) \quad . \end{aligned} \tag{2}$$

Here $\delta\mathcal{C}_x^{(1)}$ is a path consisting of two straight lines connecting $x^m(s,t)$ with $x^m(s+\Delta s, t)$ and $x^m(s+\Delta s, t)$ with $x^m(s+\Delta s, t+\Delta t)$. Similarly, $\delta\mathcal{C}_x^{(2)}$ is obtained by replacing $x^m(s+\Delta s, t)$ by $x^m(s, t+\Delta t)$ in the above. This definition can be extended to superspace in a straightforward fashion.

The right hand side of eq. (1) can of course be written as

$$g\int [\mathcal{D}v_m] \frac{k}{N_c} trT^r \left(\frac{\delta e^{iS_{YM}}}{\delta v^{(r)m}(x)}\right) U_{\mathcal{C}_{xx}} \quad . \tag{3}$$

Upon partial integration followed by the use of the completeness relation, eq. (3) produces an equation for the one- and two-loop correlators, which is the first among the infinite sequence of the Schwinger-Dyson equations:

$$\frac{\partial}{\partial x_\ell}\frac{\partial}{\partial \Sigma^{m\ell}(x)} \mathbf{W}[\mathcal{C}] = ig^2 \oint_{\mathcal{C}} dz_m << \frac{k^2}{N_c} \left(\mathrm{tr}\, U_{\mathcal{C}_{zx}}\right) \delta^{(4)}(x-z) \left(\mathrm{tr}\, U_{\mathcal{C}_{xz}}\right) >> . \tag{4}$$

Obviously one can generate the rest of the equations in the same way. We find

$$\begin{aligned} &\frac{\partial}{\partial x_\ell}\frac{\partial}{\partial \Sigma^{m\ell}(x)} << \left(\frac{k}{N_c}\right)^n \prod_{i=1}^{n} trU_{\mathcal{C}^{(i)}} >> \\ = \;& ig^2 N_c \sum_{j=1}^{n} \Theta(x \in \mathcal{C}^{(j)}) \oint_{\mathcal{C}^{(j)}} dz_m << \left(\frac{k}{N_c}\right)^{n+1} \left(\prod_{i=j+1}^{n} trU_{\mathcal{C}^{(i)}}\right) \left(\mathrm{tr}\, U_{\mathcal{C}^{(j)}_{zx}}\right) \\ & \delta^{(4)}(x-z) \left(\mathrm{tr}\, U_{\mathcal{C}^{(j)}_{xz}}\right) \left(\prod_{i=1}^{j-1} trU_{\mathcal{C}^{(i)}}\right) >> \quad . \end{aligned} \tag{5}$$

Here $\Theta(x \in \mathcal{C})$ is 1 when x belongs to the loop $\mathcal{C}$ and 0 otherwise. In the large N_c limit with $\lambda_c \equiv g^2 N_c$ kept finite, eq. (4) reduces to the closed loop

equation for $W[\mathcal{C}]$ albeit being highly singular [1]:

$$\frac{\partial}{\partial x_\ell}\frac{\partial}{\partial \Sigma^{m\ell}(x)}\mathbf{W}[\mathcal{C}] = i\lambda_c \oint_{\mathcal{C}} dz_m \mathbf{W}[\mathcal{C}_{zx}]\delta^{(4)}(x-z)\mathbf{W}[\mathcal{C}_{xz}] \quad . \tag{6}$$

We find that the above reasoning has a straightforward generalization to $N=1$ supersymmetric Yang-Mills theory in its geometric formulation on $N=1$ superspace.[3] We use the same symbols for the objects appearing above and their superspace counterparts. We closely follow the notation of [7]. The action for the supersymmetric Yang-Mills theory reads

$$S_{SYM} = \int d^4x \frac{1}{8kg^2} tr(-2i\lambda\sigma^a D_a\bar{\lambda} + D^2 - \frac{1}{2}v_{ab}v^{ab}) \quad . \tag{7}$$

This time, let the supersymmetric Wilson-loop associated with a closed loop $\mathcal{C}$ in superspace [4],[5],[6] be

$$\begin{aligned} \mathbf{W}[\mathcal{C}] &\equiv << \frac{k}{N_c} tr U_{\mathcal{C}} >> \\ \text{with} \quad U_{\mathcal{C}} &\equiv P\exp(\oint_{\mathcal{C}} \phi) \quad , \end{aligned} \tag{8}$$

where $\phi \equiv dz^M\phi_M(z) = dz^M\phi_M^{(r)}(z)iT^r$ is the connection one-form which is Lie algebra valued.[4] A solution to the proper set of constraints of flat connections on superspace together with the constraints imposed by a set of Bianchi identities provides an expression in terms of the real superfield $V(x,\theta,\bar{\theta})$ (see for instance [8],[7]):

$$(-2)\mathcal{F}_{a\dot{\alpha}} = -iW^\beta\sigma_{a\beta\dot{\alpha}} \quad , \tag{9}$$

$$\phi_\alpha = -e^{-V}D_\alpha e^V \quad , \quad \phi_{\dot{\alpha}} = 0 \quad , \quad \phi_a = \frac{i}{4}\bar{\sigma}_a^{\dot{\alpha}\alpha}\bar{D}_{\dot{\alpha}}e^{-V}D_\alpha e^V \quad , \tag{10}$$

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V}D_\alpha e^V \quad . \tag{11}$$

In calculation we employ the y coordinate ($y^a \equiv x^a + i\theta\sigma^a\bar{\theta}$) [7], in terms of which

$$\phi(y,\theta,\bar{\theta}) = dy^a\phi_a(y,\theta,\bar{\theta}) - 2id\theta\sigma^a\bar{\theta}\phi_a(y,\theta,\bar{\theta}) + d\theta^\alpha\phi_\alpha(y,\theta,\bar{\theta}) \quad . \tag{12}$$

[3]The above derivation can, of course, be presented as $0 = \int[\mathcal{D}v_m]tr\left(T^r \frac{\delta}{\delta v_m^{(r)}(x)}\left(U_{\mathcal{C}}e^{iS_{YM}}\right)\right)$. This, however, does not lead to a fruitful superspace generalization.

[4] A rescaling by factor $2g$ in the gauge potentials from the non-supersymmetric case is understood and ϕ_M is antihermitean.

Area derivative in superspace can be constructed in an entirely analogous way as in Minkowski space. For any functional $f[\mathcal{C}_{yz}]$ on superspace, the area derivative $\partial/\partial\Sigma^{ML}(x)$ can be defined in any direction including the grassmannian ones by considering

$$\begin{aligned}\Delta f[\mathcal{C}_{yz}] &\equiv f[\mathcal{C}_{yz}+\delta\mathcal{C}_x^{(1)}]-f[\mathcal{C}_{yz}+\delta\mathcal{C}_x^{(2)}]\\ &\equiv \frac{\partial f[\mathcal{C}_{yz}]}{\partial\Sigma^{ML}(x)}\frac{1}{2}\left(\frac{\partial x^M}{\partial s}\frac{\partial x^L}{\partial t}-\frac{\partial x^M}{\partial t}\frac{\partial x^L}{\partial s}\right)\Delta s\Delta t \quad . \qquad (13)\end{aligned}$$

We find

$$tr(\sigma^a_{\alpha\dot\beta}\epsilon^{\dot\beta\dot\alpha}D^\alpha\frac{\partial U_{\mathcal{C}_{yy}}}{\partial\Sigma^{a\dot\alpha}(y)})=2itr\left((\mathcal{D}W)U_{\mathcal{C}_{yy}}\right) \quad , \qquad (14)$$

where

$$\begin{aligned}\mathcal{D}W\equiv DW-\phi W+W\phi &= D^\alpha W_\alpha-\{\phi^\alpha,W_\alpha\} \qquad (15)\\ \frac{\partial U_{\mathcal{C}_{yy}}}{\partial\Sigma^{AB}(y)} &= F_{AB}(y)U_{\mathcal{C}_{yy}} \quad . \qquad (16)\end{aligned}$$

Here A,B are flat indices in superspace.

In order to convert $\sigma^a_{\alpha\dot\beta}\epsilon^{\dot\beta\dot\alpha}D^\alpha\frac{\partial}{\partial\Sigma^{a\dot\alpha}}\mathbf{W}[\mathcal{C}]$ into another expression, we compute $\mathcal{D}W$ in the W-Z gauge. For that purpose, we first find the expression of ϕ_a and that of ϕ_α in the W-Z gauge:

$$\xi\phi=\xi\sigma^a\bar\theta v_a-2i\xi\theta\bar\theta\bar\lambda+i\bar\theta\bar\theta\xi\lambda-\xi\theta\bar\theta\bar\theta D+i\bar\theta\bar\theta\xi\sigma^{ab}\theta v_{ab}-\theta\theta\bar\theta\bar\theta\xi\sigma^a D_a\bar\lambda, \qquad (17)$$

$$2i\phi_a=v_a-i\bar\lambda\bar\sigma_a\theta+i\bar\theta\bar\sigma_a\lambda-\bar\theta\bar\sigma_a\theta D+i\bar\theta\bar\sigma_a\sigma^{bc}\theta v_{bc}-\theta\theta\bar\theta\bar\sigma_a\sigma^b D_b\bar\lambda \quad . \qquad (18)$$

These are again in the y coordinate. After some amounts of algebras, we find

$$\begin{aligned}-\mathcal{D}W &= 2D+2\bar\theta\bar\sigma^a D_a\lambda-2\theta\sigma^a D_a\bar\lambda+2\theta\sigma_b\bar\theta(D_a v^{ab}-\frac{1}{2}\bar\sigma^{b\dot\beta\beta}\{\bar\lambda_{\dot\beta},\lambda_\beta\})\\ &- 2i\theta\sigma^a\bar\theta D_a D+2i\theta\theta\bar\theta\bar\sigma^b\sigma^a D_b D_a\bar\lambda+2i\theta\theta[D,\bar\theta\bar\lambda] \quad , \qquad (19)\end{aligned}$$

where $D_a\cdots\equiv[\partial_a+\frac{i}{2}v_a,\cdots]$. As $\mathcal{D}W=0$ summarizes equations of motion for $v_m,\lambda,\bar\lambda,D$, it should be that $\mathcal{D}W$ is written as a particular variation of S_{SYM} as well. We find

$$\begin{aligned}- \quad \mathcal{D}W=8g^2\Big\{&T^r\frac{\delta}{\delta D^{(r)}}+iT^r\bar\theta_{\dot\alpha}\frac{\delta}{\delta\bar\lambda^{(r)}_{\dot\alpha}}-iT^r\theta^\alpha\frac{\delta}{\delta\lambda^{\alpha(r)}}-T^r\bar\theta\bar\sigma_a\theta\frac{\delta}{\delta v_a^{(r)}}\\ + \quad &i\bar\theta\bar\sigma_a\theta D_a T^r\frac{\delta}{\delta D^{(r)}}-\theta\theta(\bar\theta\bar\sigma^b)^\alpha D_b T^r\frac{\delta}{\delta\lambda^{\alpha(r)}}+i\theta\theta\,[T^r\frac{\delta}{\delta D^{(r)}},\bar\theta\bar\lambda]\Big\}S_{SYM} \qquad (20)\end{aligned}$$

We denote this expression by $8g^2\delta S_{SYM}/\delta V_{mod}(y,\theta,\bar\theta)$.

From eq. (14) and eq. (20), we obtain

$$\begin{aligned}
&\sigma^a_{\alpha\dot\beta}\epsilon^{\dot\beta\dot\alpha}D^\alpha\frac{\partial}{\partial\Sigma^{a\dot\alpha}(y')}\mathbf{W}[\mathcal{C}]\\
=\;& -16g^2\int[\mathcal{D}v_m][\mathcal{D}\lambda][\mathcal{D}\bar\lambda][\mathcal{D}D]\frac{k}{N_c}tr\frac{\delta e^{iS_{SYM}}}{\delta V_{mod}(y',\theta,\bar\theta)}U_{\mathcal{C}}\\
=\;& 16g^2 << \frac{k}{N_c}tr\frac{\delta}{\delta V_{mod}(y',\theta,\bar\theta)}U_{\mathcal{C}} >> \quad .
\end{aligned}\tag{21}$$

We have now replaced y by y' and saved y for the integration variable. To evaluate the right hand side, we compute (again in the y coordinate)

$$\left(\frac{\delta}{\delta V_{mod}(y',\theta,\bar\theta)}\right)_i^{\ j}(\phi(y,\eta,\bar\eta))_k^{\ \ell} \quad .\tag{22}$$

The calculation is long and space permits us to present the final result only:

$$\begin{aligned}
&\left(\frac{\delta}{\delta V_{mod}(y',\theta,\bar\theta)}\right)_i^{\ j}(\phi(y,\eta,\bar\eta))_k^{\ \ell}\\
&= (T^r)_i^{\ j}\Big\{\Big(-\frac{i}{2}dy^a(\eta-\theta)\sigma_a(\bar\eta-\bar\theta)-\frac{1}{2}\delta(\eta-\theta)dy^a\bar\theta\bar\sigma^b\sigma_a\bar\eta\mathcal{D}_b \qquad (23)\\
&+\bar\eta\bar\eta d\eta(\eta-\theta)+i\delta(\eta-\theta)\bar\eta\bar\eta d\eta\sigma^a\bar\theta\mathcal{D}_a\Big)T^r\delta^{(4)}(y-y')\Big\}_k^{\ \ell}\\
&+(T^r)_i^{\ j}\frac{i}{4}\delta(\eta-\theta)\bar\eta\bar\eta dy^a\left([\bar\theta\bar\sigma_aW,T^r]\right)_k^{\ \ell}\delta^{(4)}(y-y')\\
&+\left(\frac{1}{2}dy^a\eta\sigma_a\bar\eta\theta\theta+i\theta\theta\bar\eta\bar\eta d\eta\eta\right)\Big\{\left([T^r,\bar\theta\bar\lambda]\right)_i^{\ j}(T^r)_k^{\ \ell}\\
&-(T^r)_i^{\ j}\left([\bar\theta\bar\lambda,T^r]\right)_k^{\ \ell}\Big\}\delta^{(4)}(y-y') \quad .
\end{aligned}$$

Here $\mathcal{D}_a\cdots\equiv[\partial_a-\phi_a,\cdots]$. We have repeatedly used

$$\frac{i}{2}(v_a-i\bar\lambda\bar\sigma_a\eta)=-(1+\bar\eta^{\dot\alpha}\bar D_{\dot\alpha})\phi_a\tag{24}$$

to convert the expression into the one containing the covariant derivative with respect to ϕ_a alone.

Using eq. (23) and the completeness relation, we are able to complete the right hand side of eq. (21). The last line of eq. (23) disappears after the completeness relation is used and the covariant derivative $\mathcal{D}_a$ and $\frac{i}{2}(W\sigma_a)_{\dot{\alpha}}$ are replaced respectively by the ordinary derivative ∂_a and the area derivative $\partial/\partial\Sigma^{a\dot{\alpha}}$. These are nontrivial properties revealed by our calculation. All in all, we find

$$
\begin{aligned}
& \sigma^a_{\alpha\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}} D^\alpha \frac{\partial}{\partial\Sigma^{a\dot{\alpha}}(y')}\mathbf{W}[\mathcal{C}] \\
= \;& 16g^2 \oint \left[dy^a \left(-\frac{i}{2}(\eta-\theta)\sigma_a(\bar{\eta}-\bar{\theta}) - \frac{1}{2}\delta(\eta-\theta)\bar{\theta}\bar{\sigma}^b\sigma_a\bar{\eta}\partial_b^{(y)} \right) \right. \\
+ \;& \left. \bar{\eta}\bar{\eta} d\eta(\eta-\theta) + i\delta(\eta-\theta)\bar{\eta}\bar{\eta} d\eta\sigma^a\bar{\theta}\partial_a^{(y)} \right] \\
\times \;& << \frac{k^2}{N_c}\left(trU_{\mathcal{C}_{yy'}}\right)\delta^{(4)}(y-y')\left(trU_{\mathcal{C}_{y'y}}\right) >> \\
+ \;& \frac{16g^2}{8}\oint dy^a \left(\bar{\sigma}^b\sigma_a\bar{\theta}\right)^{\dot{\alpha}} \delta(\eta-\theta)\bar{\eta}\bar{\eta} \\
\times \;& << \frac{k^2}{N_c}\left(trU_{\mathcal{C}_{yy'}}\right)\left(\frac{\overleftrightarrow{\partial}}{\partial\Sigma^{b\dot{\alpha}}}\right)\delta^{(4)}(y-y')\left(trU_{\mathcal{C}_{y'y}}\right) >> \quad , \\
\equiv \;& 16g^2 N_c \oint dy^A << \left(\frac{k}{N_c}\right)^2 \left(trU_{\mathcal{C}_{yy'}}\right)\hat{\mathcal{O}}_A\left(\frac{\partial}{\partial\Sigma(y)},\frac{\partial}{\partial y},y\right) \\
& \delta^{(4)}(y-y')\left(trU_{\mathcal{C}_{y'y}}\right) >> \qquad (25)
\end{aligned}
$$

where $A \frac{\overleftrightarrow{\partial}}{\partial\Sigma^{a\dot{\alpha}}} B = A\left(\frac{\partial}{\partial\Sigma^{a\dot{\alpha}}}B\right) - \left(\frac{\partial}{\partial\Sigma^{a\dot{\alpha}}}A\right)B$. It is remarkable that the right-hand side of this equation does not have any field dependence other than the Wilson-loop variable in superspace. Again we can immediately write down the rest of the Schwinger-Dyson equations:

$$
\begin{aligned}
& \sigma^a_{\alpha\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}} D^\alpha \frac{\partial}{\partial\Sigma^{a\dot{\alpha}}(y')} << \left(\frac{k}{N_c}\right)^n \prod_{i=1}^n trU_{\mathcal{C}^{(i)}} >> \\
= \;& 16g^2 N_c \sum_{j=1}^n \Theta(y'\in\mathcal{C}^{(j)}) \oint_{\mathcal{C}^{(j)}} dy^A << \left(\frac{k}{N_c}\right)^{n+1}\left(\prod_{i=j+1}^n trU_{\mathcal{C}^{(i)}}\right) \\
\times \;& \left(\mathrm{tr}\, U_{\mathcal{C}^{(j)}_{yy'}}\right)\hat{\mathcal{O}}_A \delta^{(4)}(y-y')\left(trU_{\mathcal{C}^{(j)}_{y'y}}\right)\left(\prod_{i=1}^{j-1} trU_{\mathcal{C}^{(i)}}\right) >> \quad . \qquad (26)
\end{aligned}
$$

When a matter chiral multiplet is present, we should add to the right hand side a term

$$
\begin{aligned}
& 16g^2 \sum_{j=1}^{n} \Theta(y \in \mathcal{C}^{(j)}) << \left(\frac{k}{N_c}\right)^n \left(\prod_{i=j+1}^{n} trU_{\mathcal{C}^{(i)}}\right) \\
\times \quad & \left(trJ(y', \theta, \bar{\theta})U_{\mathcal{C}^{(j)}_{y'y'}}\right) \left(\prod_{i=1}^{j-1} trU_{\mathcal{C}^{(i)}}\right) >> \quad . \qquad (27)
\end{aligned}
$$

Here $J(y', \theta, \bar{\theta}) \equiv i\delta S_{matter}/\delta V_{mod}(y', \theta, \bar{\theta})$ and S_{matter} is $\int d^4x \; \Phi^\dagger e^V \Phi \mid_{\theta\theta\bar{\theta}\bar{\theta}}$ for fundamental matter and $\int d^4x \; tr\Phi^\dagger e^V \Phi e^{-V} \mid_{\theta\theta\bar{\theta}\bar{\theta}}$ for adjoint matter.

In the large N_c limit (this time $\lambda_c \equiv 4g^2 N_c$), eq. (25) becomes a closed loop equation for $\mathbf{W}[\mathcal{C}]$:

$$
\begin{aligned}
& \frac{1}{4\lambda_c} \sigma^a_{\alpha\dot{\beta}} \epsilon^{\dot{\beta}\dot{\alpha}} D^\alpha \frac{\partial}{\partial \Sigma^{a\dot{\alpha}}} \mathbf{W}[\mathcal{C}] \\
= \quad & \oint \left[dy^a \left(-\frac{i}{2}(\eta - \theta)\sigma_a(\bar{\eta} - \bar{\theta}) - \frac{1}{2}\delta(\eta - \theta)\bar{\theta}\bar{\sigma}^b \sigma_a \bar{\eta} \partial_b^{(y)} \right) \right. \qquad (28) \\
+ \quad & \left. \bar{\eta}\bar{\eta} d\eta(\eta - \theta) + i\delta(\eta - \theta)\bar{\eta}\bar{\eta} d\eta \sigma^a \bar{\theta} \partial_a^{(y)} \right] \mathbf{W}[\mathcal{C}_{yy'}] \delta^{(4)}(y - y') \mathbf{W}[\mathcal{C}_{y'y}] \\
+ \quad & \frac{1}{8} \oint dy^a \left(\bar{\sigma}^b \sigma_a \bar{\theta} \right)^{\dot{\alpha}} \delta(\eta - \theta)\bar{\eta}\bar{\eta} \mathbf{W}[\mathcal{C}_{yy'}] \left(\frac{\overleftrightarrow{\partial}}{\partial \Sigma^{b\dot{\alpha}}} \right) \delta^{(4)}(y - y') \mathbf{W}[\mathcal{C}_{y'y}] \quad .
\end{aligned}
$$

Eqs. (25)–(28) are the results from our discussion.

Just as in the non-supersymmetric case, there remain a number of questions on how one can manage to extract physics results from eqs. (25), (28). These may involve the problem of renormalizations and more constructive formulations for instance. Originally we were partially motivated by the recent developments associated with the exact evaluation of the light effective action beginning with [9] and the surprising connection to integrable systems of particles [10]. A connection of the results presented in this paper with these is still remote but certainly we are now urged to strive for further developments.

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References

[1] Yu.M. Makeenko and A.A. Migdal, *Phys. Lett.* **B88** (1979) 135;

[2] J-L. Gervais, M.T. Jaekel and A. Neveu, *Nucl. Phys.* **B155** (1979) 75.

[3] Yu.M. Makeenko and P.B. Medvedev, *Nucl. Phys.* **B193** (1981) 444.

[4] S. J. Gates, *Phys. Rev.* **D16** (1977) 1727.

[5] S. Marculescu, *Nucl. Phys.* **B213** (1983) 523.

[6] S. J. Gates, M. Grisaru, M. Rocek and W. Siegel, *"Superspace" Benjamin/Cummings* (1983).

[7] J. Wess and J. Bagger, *"Supersymmetry and Supergravity"*, *Princeton University Press* (1992) 2nd edition.

[8] M. F. Sohnius, *Phys. Rep.* **128** (1985) 39.

[9] N. Seiberg and E. Witten, *Nucl. Phys.* **B426** (1994)19: *Nucl. Phys.* **B431** (1994)484. See K. Intriligator and N. Seiberg hep-th 9509066 for more references.

[10] A.Gorsky,I.Krichever,A.Marshakov,A.Mironov and A.Morozov *Phys.Lett.***355B**(1995)466, hep-th/9505035; E.Martinec and N.Warner, hep-th/9509161; T.Nakatsu and K.Takasaki, hep-th/9509162; R.Donagi and E.Witten, hep-th/9510101; T.Eguchi and S.K.Yang, hep-th/9510183;E.Martinec, hep-th/9510204; H.Itoyama and A.Morozov, hep-th/9511126; hep-th/9512161.